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Consideration of tip speed limitations in preliminary analysis of minimum COE wind turbines

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Abstract.

A relation between Cost Of Energy, *COE*, maximum allowed tip speed, and rated wind speed, is obtained for wind turbines with a given goal rated power. The wind regime is characterised by the corresponding parameters of the probability density function of wind speed. The non-dimensional characteristics of the rotor: number of blades, the blade radial distributions of local solidity, twist angle, and airfoil type, play the role of parameters in the mentioned relation. The *COE* is estimated using a cost model commonly used by the designers. This cost model requires basic design data such as the rotor radius and the ratio between the hub height and the rotor radius. Certain design options, DO, related to the technology of the power plant, tower and blades are also required as inputs. The function obtained for the *COE* can be explored to find those values of rotor radius that give rise to minimum cost of energy for a given wind regime as the tip speed limitation changes. The analysis reveals that iso-*COE* lines evolve parallel to iso-radius lines for large values of limit tip speed but that this is not the case for small values of the tip speed limits. It is concluded that, as the tip speed limit decreases, the optimum decision for keeping minimum *COE* values can be: a) reducing the rotor radius for places with high weibull scale parameter or b) increasing the rotor radius for places with low weibull scale parameter.

1. Introduction

Studies on upscaling are fundamental for the design of optimum large wind turbines, [1]. The analysis of the cost of energy in terms of general design parameters, such as rotor radius, R , and the ratio between hub height and rotor radius, h/R , is normally undertaken at the beginning of the design optimisation process. The influence of design options (DO) such as gearbox/generator configuration, power regulation, blade and tower technology or operation and maintenance strategy, play a crucial role when taking preliminary design decisions oriented to minimise the *COE*, such as the size of the rotor for a given rated power, P_R , [2]. When the tip speed is limited, (for instance, due to noise restrictions) the annual energy production, *AEP*, is affected, and the cost of energy changes, likely producing a change in the value of the rotor radius that minimises the *COE* for a given wind regime, [3]. A way to analyse this effect is presented below.

In the present work, it is assumed that the nondimensional geometry of the rotor is known. This means that, after a proper optimization process, the number of blades, the radial



distributions of local solidity, twist angle, and airfoil type, have been determined. From this set of data is possible to determine the power coefficient, C_P , as a function of the tip speed ratio, $\lambda = \Omega R V^{-1}$ (Ω is the rotational speed and V is the wind speed) and the blade control angle, θ_C , as $C_P(\lambda, \theta_C)$, and from this function, the optimum values for the tip speed ratio, λ_{op} , and control angle of the blade, θ_{Cop} , that produce maximum power coefficient, C_{Pmax} , can be obtained, see [4] and [5].

If there is not any limitation for the tip speed, the rated power can be reached with λ_{op} and θ_{Cop} , and, therefore, with C_{Pmax} ; and the power curve, $P(V)$, of the wind turbine can be expressed in two parts depending of the wind speed interval. In the first interval, $V_{in} < V \leq V_{R0}$, where V_{in} and V_{R0} are the cut-in and rated wind speed respectively, the tip speed ratio is λ_{op} and the control angle of the blade is θ_{Cop} , therefore, the power coefficient is maximum and the power curve is expressed as

$$P(V) = \frac{1}{2} \rho \pi R^2 V^3 C_{Pmax} \eta_M \eta_E, \quad (1)$$

where η_M and η_E are mechanical and electrical performances. In the second interval, $V_{R0} < V \leq V_{out}$, V_{out} is the cut-out wind speed; the rotational speed is kept constant, $\Omega = \Omega_R = \lambda_{op} V_{R0} R^{-1}$, the tip speed ratio is $\lambda = \lambda_{op} V_{R0} V^{-1}$ and the control angle of the blade is changed as the wind speed increases following a proper law $\theta_C(V)$ to keep the power constant and equal to P_R . In this second interval the power curve is simply $P(V) = P_R$. If the power curve (1) is particularised for $V = V_{R0}$, then a relation between the rated wind speed and the rotor radius can be obtained as

$$R = \left(\frac{2P_R}{\rho \pi \eta_E \eta_M C_{Pmax}} \right)^{\frac{1}{2}} V_{R0}^{-\frac{3}{2}}, \quad (2)$$

indicating, as is well known, that, for the same nondimensional design, rotors with larger radius reach the rated power at lower rated wind speeds. However, if a limitation for the tip speed is applicable, so there is a maximum tip speed, $(\Omega R)_{max} < (\Omega R)_R = \lambda_{op} V_{R0}$, the rated power cannot be reached with λ_{op} , since the tip speed must be kept constant when the wind speed reaches a value $\lambda_{op}^{-1} (\Omega R)_{max} = V_{\Omega_{max}} < V_{R0}$. From this velocity till the new rated velocity, V_R , the control angle of the blade is normally kept equal to the optimum, $\theta_C = \theta_{Cop}$, and, therefore, the wind turbine is operated as a constant velocity-stall regulated one. In this case the power curve of the wind turbine can be expressed in three parts depending on the considered wind speed interval. In the first interval, $V_{in} < V \leq V_{\Omega_{max}}$, the wind turbine operates with λ_{op} and θ_{Cop} , and, therefore, with C_{Pmax} ; and the power curve for this first part is expressed exactly as in (1). In the second part, corresponding to the wind speed interval $V_{\Omega_{max}} < V \leq V_R$, the tip speed ratio is $\lambda = (\Omega R)_{max} V^{-1}$, the control angle of the blade is θ_{Cop} and the power curve is written

$$P(V) = \frac{1}{2} \rho \pi R^2 V^3 C_P \left[\frac{(\Omega R)_{max}}{V}, \theta_{Cop} \right] \eta_M \eta_E, \quad (3)$$

and, finally, for the third interval, $V_R < V \leq V_{out}$, is again $\lambda = (\Omega R)_{max} V^{-1}$ but the control angle of the blade follows a proper law, $\theta_C(V)$ to guarantee that $P(V) = P_R$. Both types of power curves, without and with limitation of the rotational speed are schematised in figure 1.

If the expression (3) is particularised for the new rated conditions, a relation between the rotor radius, R , the new rated wind speed, V_R and the maximum tip speed, $(\Omega R)_{max}$ can be written as

$$P_R - \frac{1}{2} \rho \pi R^2 V_R^3 C_P \left[\frac{(\Omega R)_{max}}{V_R}, \theta_{Cop} \right] \eta_M \eta_E = 0. \quad (4)$$

This relation allows to obtain the radius of the wind turbine in terms of the new rated wind speed, V_R , and the maximum tip speed, $(\Omega R)_{max}$. Observe that the relation (4) is conceptually equivalent to (2), but a significant difference exists since, when there is no tip speed limitation,

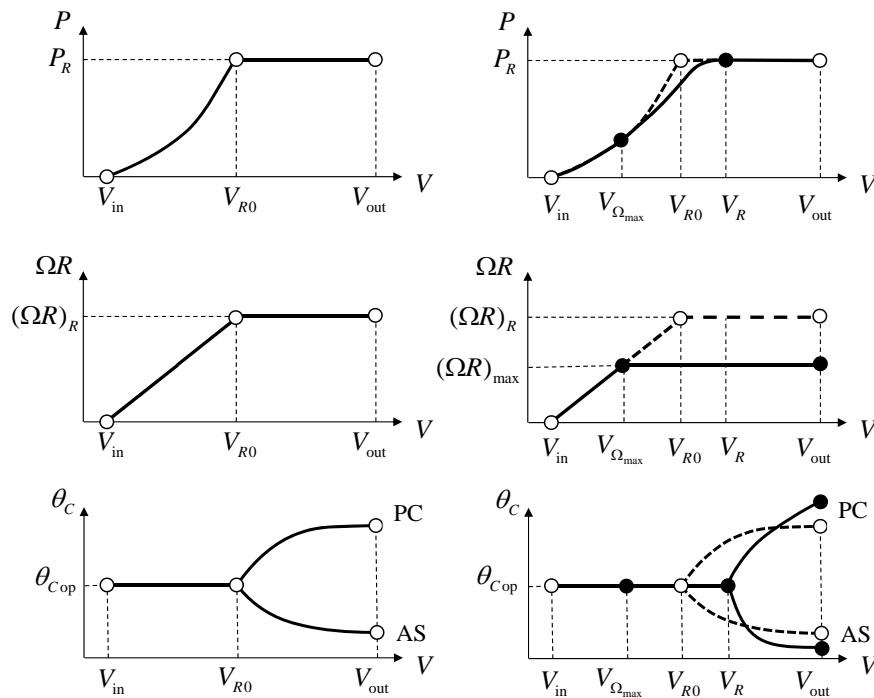


Figure 1. Power curve, tip speed and control angle of the blade versus wind speed for a case without limitation of the rotational speed (left) and with limitation of the rotational speed (right). PC: Pitch controlled, AS: Active stall.

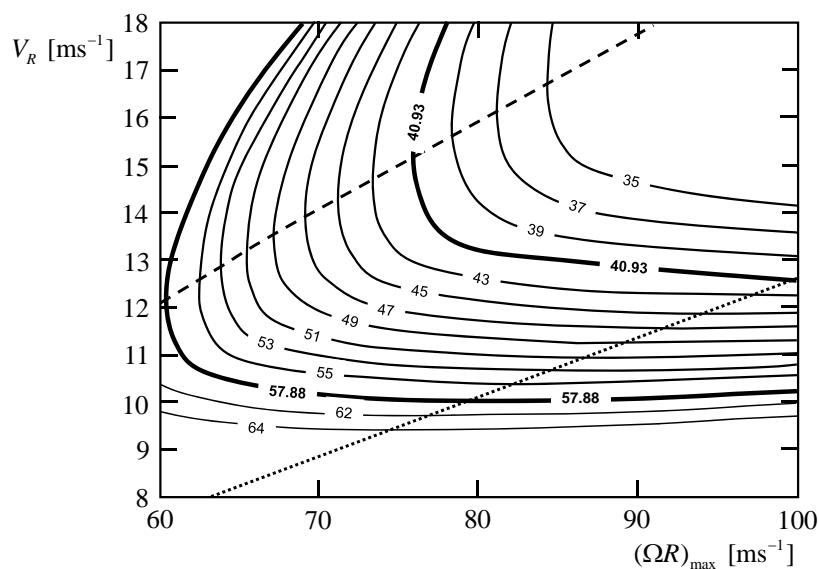


Figure 2. Rated wind speed, V_R , versus maximum tip speed, $(\Omega R)_{max}$. Left: for constant values of rotor radius, R [m] (thin solid line). $P_R = 3 \times 10^6 \text{ W}$, $\eta_M = \eta_E = 0.95$, 3 blades. The thick solid lines represent the radius $R_{min/max}$ determined for the given P_R and $SP_{max/min} = 350, 400 \text{ Wm}^{-2}$. The dashed line represents the upper limit for the region $[(\Omega R)_{max}, V_R]$ associated to the existence of $P > P_R$, the dotted line represents the lower limit for the region $[(\Omega R)_{max}, V_R]$ associated to the existence of $(\Omega R)_{max} > \lambda_{op} V_{R0}$.

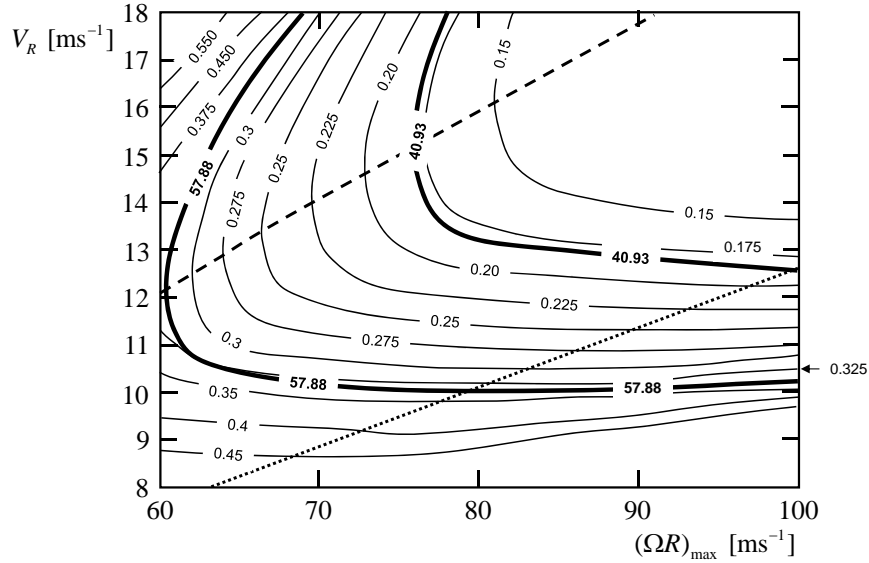


Figure 3. Rated wind speed, V_R , versus maximum tip speed, $(\Omega R)_{\max}$ for constant values of Capacity Factor, FC , (thin solid line). The case study is the same as the analyzed in figure 3. $V_{\text{in}} = 3 \text{ ms}^{-1}$, $V_{\text{out}} = 25 \text{ ms}^{-1}$, $c = 7 \text{ ms}^{-1}$, $k = 2.5$. The thick solid lines represent the radius $R_{\text{min/max}}$ determined for the given P_R and $SP_{\text{max/min}} = 350, 400 \text{ Wm}^{-2}$. The dashed line represents the upper limit for the region $[(\Omega R)_{\max}, V_R]$ associated to the existence of $P > P_R$, the dotted line represents the lower limit for the region $[(\Omega R)_{\max}, V_R]$ associated to the existence of $(\Omega R)_{\max} > \lambda_{\text{op}} V_R$.

and once P_R and R are fixed, there is a unique possible value of the rated wind speed V_{R0} . However, for given values of P_R and R , if the tip speed is limited to $(\Omega R)_{\max}$, the new rated wind velocity V_R depends on the tip speed limit $(\Omega R)_{\max}$. To illustrate this fact, the equation (4) is represented in the figure 2 for a case study corresponding to a wind turbine with $P_R = 3 \times 10^6 \text{ W}$. In the figure a maximum/minimum statistical feasible radii have been plotted as technological limits. These values for the radius have been obtained from real data of specific power, SP , provided by wind turbine manufacturers, as $R_{\text{max,min}} = P_R^{0.5} (\pi SP_{\text{min,max}})^{-0.5} = 57.88 \text{ m}/40.93 \text{ m}$. Observe that for a given radius, a reduction in the maximum tip velocity means an increment in the rated wind speed.

Once a point $[(\Omega R)_{\max}, V_R]$ is fixed in figure 2, the rotor radius is fixed, and therefore the three parts of the power curve are determined. If a probability density function for the wind speed is known, for instance a Weibull distribution, $f(V; c, k)$, where c and k are the scale and shape parameters respectively, it is possible to obtain the annual energy production as $AEP = T \times \bar{P}[(\Omega R)_{\max}, V_R, c, k]$, being T the number of seconds in one year and \bar{P} the averaged power given by

$$\bar{P}[(\Omega R)_{\max}, V_R, c, k] = \int_{V_{\text{in}}}^{V_{\text{out}}} P[V; (\Omega R)_{\max}, V_R] f(V; c, k) dV. \quad (5)$$

The AEP is affected by the tip speed limitation as it is shown in figure 3. In this figure, the capacity factor, $FC = \bar{P}/P_R$ is presented for the case study being analysed. It is observed that, within the realizable limits, for a given rated wind speed, the capacity factor increases as $(\Omega R)_{\max}$ decreases, since as it is shown in figure 2 the rotor radius increases (this is required for achieving P_R at V_R). Although this is the main effect in AEP , it can be also seen, as a secondary

effect of reducing $(\Omega R)_{\max}$, that the iso-radius lines evolve almost parallel to iso-capacity factor lines for large values of $(\Omega R)_{\max}$, however, when $(\Omega R)_{\max}$ is small, iso-radius lines evolve towards the region of lower capacity factor (lower AEP). This can be observed in figure 3 where the line for $R = 57.88\text{m}$ cuts the line $FC = 0.325$ from larger to lower values of FC . This result quantifies the fact that the lesser the value $(\Omega R)_{\max}$ the smaller the value $V_{\Omega\max}$, being smaller the interval of wind speeds in which the wind turbine operates at $C_{P\max}$.

2. The cost model

The cost model applied here has been developed by NREL. A detailed description of it can be found in [6]. The cost of energy, COE , is modelled in the mentioned reference as

$$COE = \frac{FCR \times ICC + O\&M + LRC}{AEP} + LLC, \quad (6)$$

where FCR is the Fixed Charge Rate, ICC is the Initial Capital Cost, $O\&M$ are the Levelized Operations and Maintenance Cost, LRC is the Levelized Replacement/Overhaul Cost and LLC is the Land Lease Cost. All the terms in expression (6) are expressed, in the mentioned reference, as functions of the rated power, P_R , rotor radius, R , and the ratio h/R . Additionally, some design options, DO , regarding the technology of the blade, tower and the drive-train must be selected (see [6] for details). If the AEP is expressed in terms of the averaged power given by (5) and it is substituted in (6), the COE can be expressed as

$$COE = COE[(\Omega R)_{\max}, V_R; c, k; DO, h/R], \quad (7)$$

therefore, as a function of the rated wind speed, the tip speed limit, the design options required by the cost model, the ratio of the hub height to the rotor radius and, finally, the parameters of the probability density function of the wind speed. The expression (7) is shown in the figure 6 for the case study being analysed. In the context of the presented model, once the rated power is fixed, the maximum tip speed, $(\Omega R)_{\max}$, affects the COE because, for a given AEP , the rotor radius, R , must increase when $(\Omega R)_{\max}$ decreases producing an increment of the initial capital cost, or for a given radius, lower values of $(\Omega R)_{\max}$ give rise to lower values of AEP .

For the case study shown in figure 4 ($c = 7\text{ms}^{-1}$, $k = 2.5$), the region of minimum COE is located roughly on the line corresponding to the maximum radius based on the specific power criterion (57.88m). It can be observed that iso- COE lines evolve parallel to iso-radius lines in the region of large values of $(\Omega R)_{\max}$. However, for small values of $(\Omega R)_{\max}$, the iso- COE lines cut the iso-radius lines from lower the region of values of radius to the region of larger ones. This fact indicates that the increment in AEP associated to the increment in radius has a larger influence in the COE than the associated to the increment in the cost of the wind turbine. This behaviour depends on the wind regime, since depending on the values of the scale and shape parameters, c and k , the COE function given in (7) changes substantially, because the AEP is strongly affected. If the figure 4 is reproduced for a higher value of the scale parameter, let us say $c = 10.5\text{ms}^{-1}$, see figure (5), it is observed that, as expected, the values of COE are reduced and the region of minimum COE has moved to a region of lower radius. It is remarked that, in this case, for the region of interest (low COE and high FC), again the iso- COE lines evolve parallel to the iso-radius lines in the region of large $(\Omega R)_{\max}$ but, in this case, keeping low COE values in the region of low $(\Omega R)_{\max}$ requires to reduce the rotor radius, indicating that for large values of c and low values of $(\Omega R)_{\max}$ the reduction in the cost of the turbine associated to a reduction in R influences more the COE than the corresponding reduction in AEP .

3. Conclusions

For a fixed rated power, the tip speed limitation influences the relation between the rotor radius and the rated wind speed. Lower tip speed limits give rise to higher rated velocities for the same

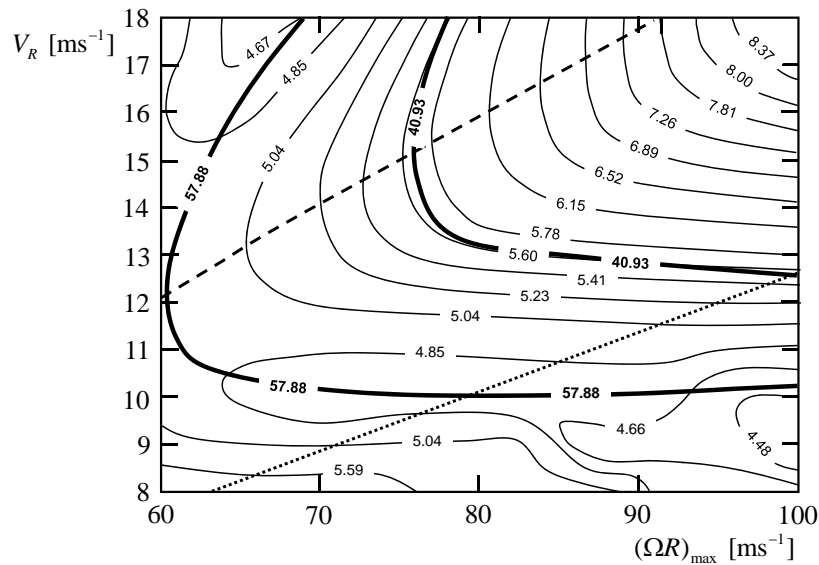


Figure 4. Rated wind speed, V_R , versus maximum tip speed, $(\Omega R)_{\max}$ for constant values of COE [c€/kWh] (thin solid line). The case study is the same as the analyzed in figures 2 and in the region of small values of the tip speed limit, keeping a constant COE implies to reduce the rotor radius. Minimum COE regions are identified. The shape and location of these region strongly depend on the weibull parameters3, but for $c = 7 \text{ ms}^{-1}$, $k = 2.5$. The DO are: Blade technology: “Base line”, Plant technology: “3 stage”, Tower technology: “Baseline”. see [6] for details on the design options, DO.

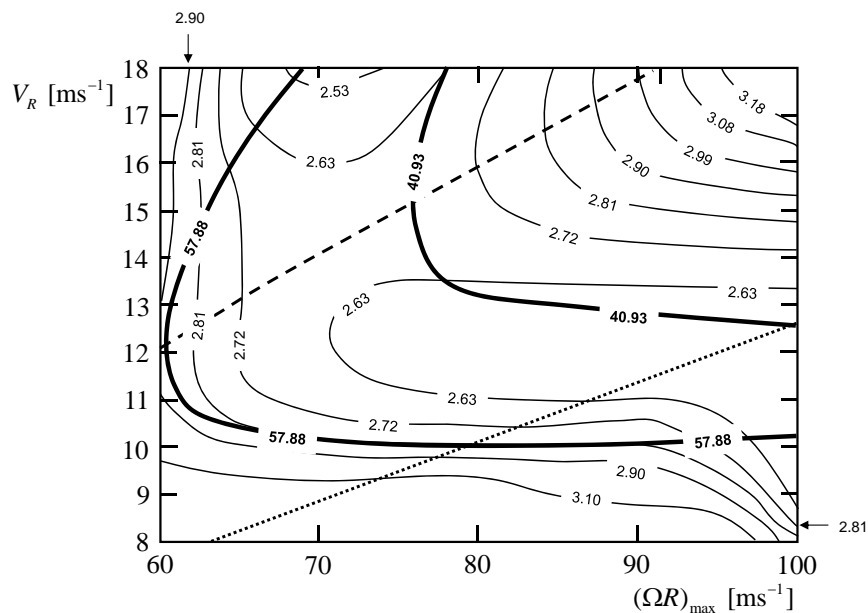


Figure 5. Rated wind speed, V_R , versus maximum tip speed, $(\Omega R)_{\max}$ for constant values of COE [c€/kWh] (thin solid line). The case study is the same as the analyzed in figures 2 and 3. $h/R = 1.25$. The DO are: Blade technology: “Base line”, Plant technology: “3 stage”, Tower technology: “Baseline”.

rotor radius. For high values of the tip speed limit, the lines of constant capacity factor evolve parallel to the iso-radius lines as the tip speed limit decreases. However, in the region of low speed limits, the lines of constant radius cross the iso- FC lines from the region of larger values of capacity factor to the region of lower values of this parameter. The iso- COE lines are parallel to the iso-radius lines for high tip speed limits, however, this is not the case for small values of the tip speed limits. It is concluded that, as the tip speed limit decreases, the optimum decision for keeping minimum COE values can be: *a)* reducing the rotor radius for places with high weibull scale parameter or *b)* increasing the rotor radius for places with low weibull scale parameter.

References

- [1] Sieros G, Chavariopoulos P, Sørensen J, Bulder B and Jaimeson P 2012 *Wind Energy* **15** 3–17
- [2] Collicutt G and Flay R 1996 *J. Wind Eng. Ind. Aerodyn.* **61** 87–97
- [3] Yeow T, Cuerva A, Pérez-Alvarez J and Munuera A 2009 *5th PhD Seminar on Wind Energy in Europe, 2009. Durham.*
- [4] Burton T, Sharpe D, Jenkins N and Bossanyi E 2001 *Wind Energy Handbook* (Chichester: John Wiley and Sons)
- [5] Cuerva A, Roura M, Sanz-Andrés A, López-García O and Franchini S 2009 *European Wind Energy Association, March, 16-19 2009, Marseille 2009*
- [6] Fingersh L, Hand M and Laxson A 2006 Wind turbine design cost and scaling model Tech. Rep. NREL/TP-500-40566 National Renewable Energy Laboratory